Work on Multi-label Classification

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Multi-label Classification: An Introduction

Data instance:

- Binary Classification: e.g. is this a beach? $\in \{\texttt{No}, \texttt{Yes}\}$

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- Multi-class Classification: e.g. what is this? $\in \{\texttt{Beach}, \texttt{Forest}, \texttt{City}, \texttt{People}\}$
- Multi-label Classification: e.g. which of these? $\subseteq \{\texttt{Beach}, \texttt{Forest}, \texttt{City}, \texttt{People}\}$

Multi-label classification is the supervised classification task where each data instance may be associated with *multiple* class labels.

Multi-label Classification: An Introduction

- Input space: $\mathcal{X} = \mathbb{R}^d$
- Instance $\mathbf{x} = [x_1, \ldots, x_d] \in \mathcal{X}$
- Class labels: $\mathcal{L} = \{1, 2, \ldots, L\}$
- Label space: $\mathcal{Y} = \{0, 1\}^L$
- Labelset: $\mathbf{y} = [y_1, \ldots, y_L] \in \mathcal{Y}$; $y_j = 1$ if jth label relevant to $\mathbf{x}$; else 0)
- Training set: $\{(\mathbf{x}_i, \mathbf{y}_i) | i = 1, \ldots, N\} \subset (\mathcal{X} \times \mathcal{Y})$
- Classification: $h : \mathcal{X} \rightarrow \mathcal{Y}$
- Prediction: $\hat{\mathbf{y}} = h(\mathbf{x})$
- Evaluation:
    - compare each $\hat{\mathbf{y}}_i$ with each $\mathbf{y}_i$ (*labelset accuracy*); OR
    - compare each $\hat{y}_j$ with each y_j (*label accuracy*).

Example Applications

Multi-label classification is relevant to many domains:

- Text
  - text documents → subject categories
  - e-mails → labels
  - medical description of symptoms → diagnoses
- Vision
  - images/video → scene concepts
  - images/video → objects identified/recognised
- Audio
  - music → genres / moods
  - sound signals → events / concepts
- Bioinformatics
  - genes → biological functions
- Sensor Networks / Robotics
  - sensor inputs → states / error diagnoses

Methods for Multi-label Classification

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- Binary Relevance method (BR) One binary classifier for each label:
  - $\hat{\mathbf{y}} \equiv [\hat{y}_1, \ldots, \hat{y}_L] = [h_1(\mathbf{x}), \ldots, h_L(\mathbf{x})]$
    - where each $y_j \in \{0, 1\}$
  - simple; flexible; fast
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- Label Powerset method (LP): One multi-class classifier; one class for each *labelset*:
  - $\hat{\mathbf{y}} \equiv \hat{c} = h(\mathbf{x})$
    - where class $\hat{c} \in \mathcal{C}$, $\mathcal{C} = 2^{\mathcal{L}}$ (in practice $\mathcal{C} = \mathrm{distinct}(\{\mathbf{y}_1, \ldots, \mathbf{y}_N\})$)
  - models label dependencies; good accuracy
  - *but* high complexity ($\min(N, 2^L)$); overfitting; difficult for incremental classification

Ensembles Of Classifier Chains

Efficiently modelling label dependencies in a BR-like way using Classifier Chains (CC) [Read et al., 2009].

- BR: $\hat{\mathbf{y}} \equiv [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_L] = [h_1(\mathbf{x}), h_2(\mathbf{x}), \ldots, h_L(\mathbf{x})]$
- CC: $\hat{\mathbf{y}} \equiv [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_L] = [h_1(\mathbf{x}), h_2(\mathbf{x}, \hat{y}_1), \ldots, h_L(\mathbf{x}, \hat{y}_1, \hat{y}_2, \ldots, \hat{y}_{L-1})]$

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Predictive performance depends on how labels are indexed in $\mathbf{Y}$. We used an *Ensemble of (random) Classifier Chains* (ECC):

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A probabilistic interpretation [Cheng et al., 2010]:

$$P_{\mathbf{x}}(\mathbf{y}) = P_{\mathbf{x}}(y_1) \prod_{j=1}^{L} P_{\mathbf{x}}(y_j | y_1, \ldots, y_{j-1})$$

- Improves on CC, but not ECC; complex (searches all 2^L combinations)

Labelset Mapping

An issue with classifier chains:

- high memory use (large L = large feature space).

New method: k-Labelset Mapping (kLM):

- k-Nearest Neighbors in the label space of BR:
  - $\hat{\mathbf{w}} = \mathbf{h}(\mathbf{x})$, $\mathbf{w} \in \mathbb{R}^L$
  - $d_i = \texttt{euclidean\_dist}(\mathbf{y}_i, \hat{\mathbf{w}})$ for $\mathbf{y}_i \in \texttt{distinct}(\{\mathbf{y}_1, \ldots, \mathbf{y}_N\})$
  - map $\hat{\mathbf{w}}' = \mathrm{avg}(\mathbf{y}_1, \ldots, \mathbf{y}_k)$ from smallest $d_1, \ldots, d_k$
  - map $\hat{\mathbf{y}} = f_t(\hat{\mathbf{w}}')$
- e.g. $[0.9, 0.0, 0.5, 0.8] \mapsto \{[1, 0, 0, 1]_{0.8}, [1, 0, 0, 0]_{1.4}, [0, 0, 0, 1]_{1.6}\} \mapsto [0.6, 0.0, 0.0, 0.6] \mapsto [1, 0, 0, 1]$
- Models label dependency like LP, fast like BR
- Faster than, and competitive with CC

But, same story: different classifiers perform better on different datasets:

- BR: fast, best when labels are independent
- LP-based, (e.g., kLM): good when only a few *distinct* labelsets (indicating *label interdependence*)
- CC: performs well overall by modelling label interdependence approximately (but doesn't specialise)

Label dependencies are the biggest influencing factor on performance, directly and indirectly.

Label Dependence — A Closer Look

It was always clear that:

- there are *dependencies* (i.e., correlations) between labels; and
- modelling these dependencies *improves predictive performance*; but
- is inherently *expensive* ($\frac{L(L-1)}{2}$ *pairwise*, 2^L *all*).

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- only map dependent labels with kLM
- etc.

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In multi-label data [Read, 2010, Dembczyński et al., 2010]:

- Unconditional dependence: *correlations* in $\mathbf{Y}$.
- Conditional dependence: dependencies in $\mathbf{Y}$ given a specific $\mathbf{x} \in \mathcal{X}$

Unconditional (In)dependence

- A vector of labels $\mathbf{Y}$ is is **unconditionally** ***L*-independent** if:

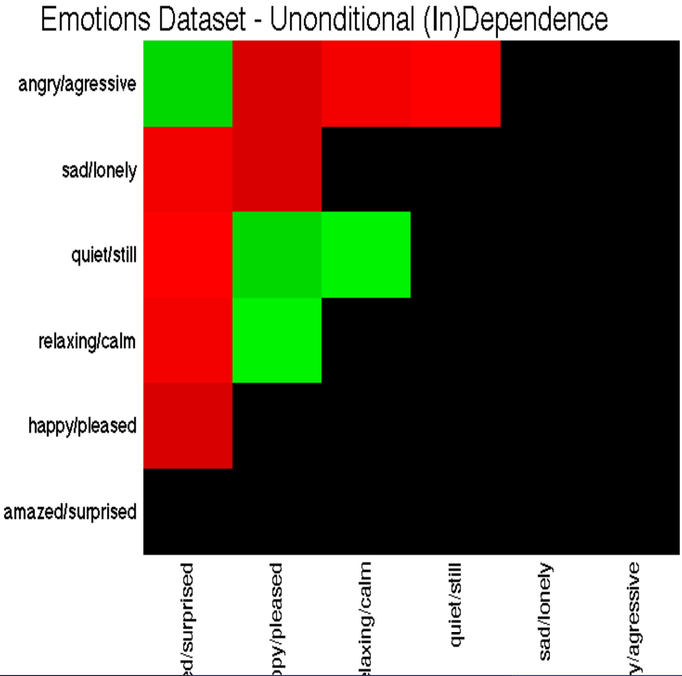
$$p(\mathbf{Y}) = \prod_{j=1}^{L} p(Y_j)$$

- Can measure with *Mutual information*,

$$I(Y_j; Y_k) = \sum_{y_j \in \{0,1\}} \sum_{y_k \in \{0,1\}} p(y_j, y_k) \log\left(\frac{p(y_j, y_k)}{p_1(y_j) p_2(y_k)}\right)$$

- or *Pearson's Correlation Coefficient*

$$P_{Y_j, Y_k} = \frac{\mathrm{cov}(Y_j, Y_k)}{\sigma_{Y_j} \sigma_{Y_k}}$$


Emotions Dataset - Unonditional (In)Dependence
angry/agressive
sad/lonely
quiet/still
relaxing/calm
happy/pleased
amazed/surprised
ed/surprised
ppy/pleased
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Conditional (In)dependence

Label dependence taking into account a specific instance: $P(y_j|y_k, \mathbf{x})$. Can use, for example:

- *Four-class pairWise classification* method (FW):
  - Models $y_{jk} \in \{00, 01, 10, 11\}$ for *each pair* of labels $y_{1 \leq j < k \leq L}$
  - prediction $\hat{\mathbf{y}} =$ average label votes for each y_j (and use a threshold).

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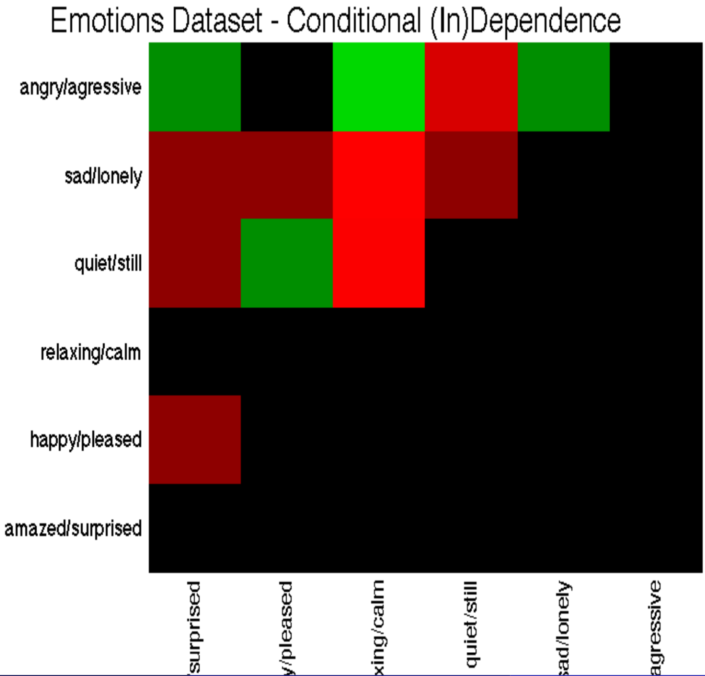
and compare to performance vs. BR to measure significant dependence.

Table: On synthetic data with strong conditional dependence and independence.

	Conditional Dependence				Conditional Independence			
	FW	BR	LP	CC	FW	BR	LP	CC
Subset Acc.	**0.77**	0.70	0.69	0.74	0.84	**0.89**	0.59	0.88
Labelset Acc.	**0.45**	0.38	0.38	0.43	0.59	**0.61**	0.43	0.60
Label Acc.	**0.94**	0.92	0.91	**0.94**	0.97	**0.98**	0.90	**0.98**

- FW (also CC) best at modelling conditional dependence.
- If complete independence, BR is best choice.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Conditional[1] and unconditional dependence and independence.

label pair	conditional.	uncond.
{amazed-suprised, happy-pleased}	-0.565 ± 0.33	0.0
{amazed-suprised, relaxing-clam}	3.806 ± 1.028	0.476
{amazed-suprised, quiet-still}	-0.422 ± 0.020	0.59
{amazed-suprised, sad-lonely}	0.566 ± 0.742	0.362
{amazed-suprised, angry-aggresive}	-2.258 ± 2.071	0.126
{happy-pleased, relaxing-clam}	2.26 ± 5.283	0.028
{happy-pleased, quiet-still}	2.534 ± 0.078	0.225
{happy-pleased, sad-lonely}	3.512 ± 10.79	0.427
{happy-pleased, angry-aggresive}	1.685 ± 2.939	0.369
{relaxing-clam, quiet-still}	0.986 ± 0.021	0.455
{relaxing-clam, sad-lonely}	-0.281 ± 0.082	0.19
{relaxing-clam, angry-aggresive}	1.554 ± 3.485	0.8
{quiet-still, sad-lonely}	0.425 ± 0.515	0.547
{quiet-still, angry-aggresive}	1.276 ± 9.072	0.395
{sad-lonely, angry-aggresive}	1.13 ± 1.321	0.215

[1] 15×2 CV eval(FW,$\mathcal{D}$)−eval(BR,$\mathcal{D}$) on training set $\mathcal{D}$

Preliminary Results; Current & Future Work

For each pair Y_j, Y_k, if $I(Y_j; Y_k) > 0.4$, train FW ($y_{jk} \in \{00, 01, 10, 11\}$) for this pair, else train (BR $y_{jk} \in \{0, 1\}^2$). Sum votes for each label (and use a threshold) to get a labelset prediction $\hat{\mathbf{y}}$:

Table: Comparing performance on the *Emotions* dataset

	BR	FW	BR/FW
Subset Accuracy	0.46	0.54	**0.57**
Labelset Accuracy	0.21	0.27	**0.29**
Label Accuracy	0.78	**0.79**	**0.79**

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Current/Future Work:

- Combining BR with FW, LP, CC, etc.
- Improving kLM, classifier chains (CC), etc.
- Learning Conditional Random Fields (CRFs).
- Feature selection/reduction.

Sidetrack: Feature Selection

So far, many of the best/popular methods are based on:

- binary relevance; or
- LP-*subsets*.

For example predicting the relevance of label y_1 under BR:

$$\hat{y}_1 = h_1(\mathbf{x})$$

Are *all* features in $\mathbf{X}$ relevant to predicting only Y_1?

- Answer so far: often not!
  - For BR, accuracy peaks using the top 30-50% of features.
- Big savings to be had if we can figure out which feature subset to use (efficiently).

Multi-label Classification in Datastreams

Initial work ([Read et al., 2010]; from [Read, 2010] & [Bifet et al., 2009]) on *instance*-incremental multi-label classification in datastreams (data arriving continuously and rapidly; concept drift expected):

- Adaptive Ensembles of Classifier Chains (ECC)
  - Hoeffding trees as base-classifiers
  - reset classifiers based on current performance / concept drift
- Multi-label Hoeffding Tree:
  - Label Powerset method (LP) at the leaves
  - an ensemble strategy to deal with concept drift
- Experiments/Results:
  - generating synthetic multi-label data streams
  - setting a benchmark on real-world and synthetic data

Current / Future Work:

- beating the benchmark
- modelling label dependencies incrementally, and drift within
- incremental feature selection

Summary / Other areas of interest

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- multi-label methods
- dependencies in multi-labeled data
- more accurate / efficient methods using these dependencies
- special considerations for data streams

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Other areas of interest:

- Distributed Algorithms for Tracking in Sensor Networks
  - testbed implementation
  - particle filters
  - machine learning
  - multi-label classification

The End

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