Aalto University
School of Science
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On the behavior of tile assembly system at high temperatures

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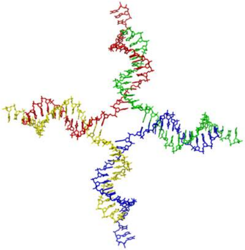
Molecular self-assembly

Engineering Goal

Practice of DNA self-assembly

In 1982, Seeman proposed the following laboratory implementation of DNA tile called holiday junction [Seeman 1982].



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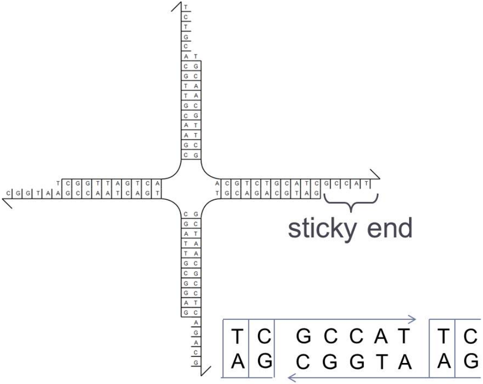
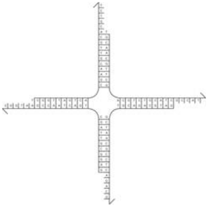


Practice of DNA self-assembly



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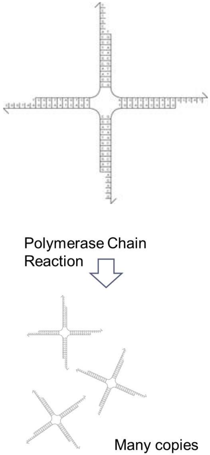

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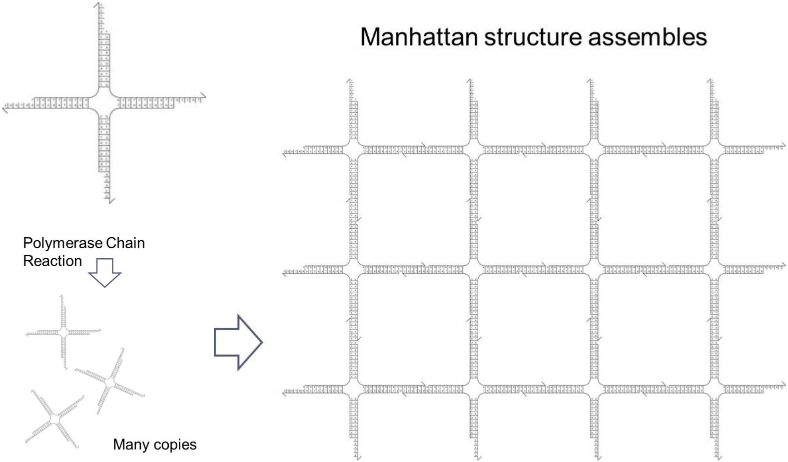

Theory of algorithmic self-assembly

Erik Winfree asked: what if . . .

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- there are more than one tile types, with different sticky ends?

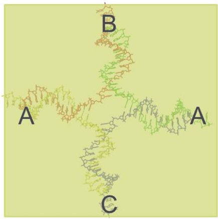
Theory of algorithmic self-assembly

Erik Winfree asked: what if …

- there are more than one tile types, with different sticky ends?
- some sticky ends are weak?

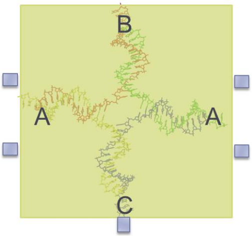
Abstract Tile-Assembly Model (aTAM) [Winfree 1998]

- A DNA holiday junction is abstracted to be a unit square tile.
- Sticky ends at each of its 4 sides are abstracted to be glues taken from a set Σ



Abstract Tile-Assembly Model (aTAM) [Winfree 1998]

- A DNA holiday junction is abstracted to be a unit square tile.
- Sticky ends at each of its 4 sides are abstracted to be glues taken from a set Σ
- A function $g : \Sigma \to \mathbb{N}_0$ assigns each label a strength.



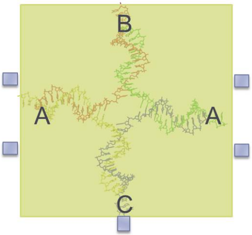
$g(\mathrm{A}) = 2, g(\mathrm{B}) = 0, g(\mathrm{C}) = 1$

Abstract Tile-Assembly Model (aTAM) [Winfree 1998]

A tile assembly system (TAS) is a 4-tuple (T, σ, g, τ), where

- T a set of tile types
- σ a special tile type called seed
- g glue strength function
- τ a positive integer threshold called temperature

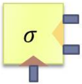
Assembly begins with a single copy of seed σ.



$g(\mathrm{A}) = 2, g(\mathrm{B}) = 0, g(\mathrm{C}) = 1$

An example: self-assembly of $n \times n$ square using $n + 3$ tile types at temperature 2

A tile can attach to an assembly if it binds with total strength at least τ (temperature). In this example, $\tau = 2$.



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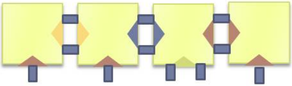

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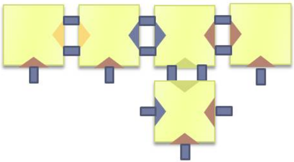

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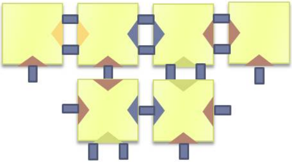

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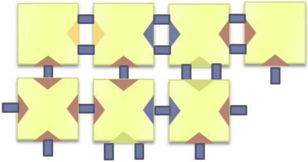

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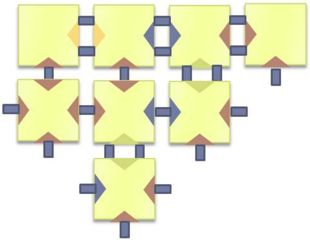

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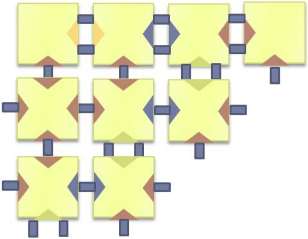

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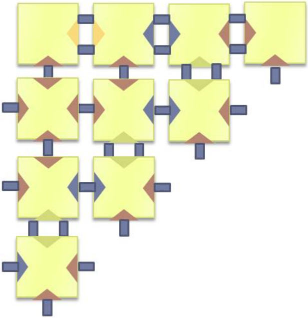

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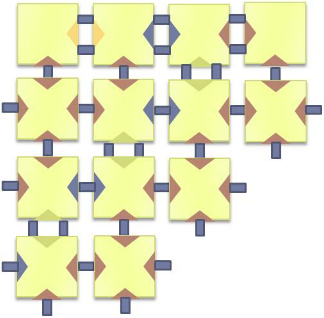

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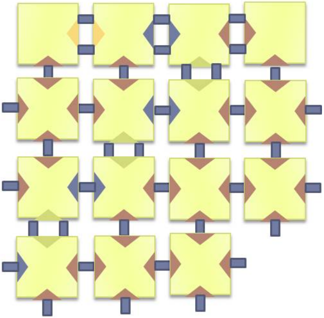

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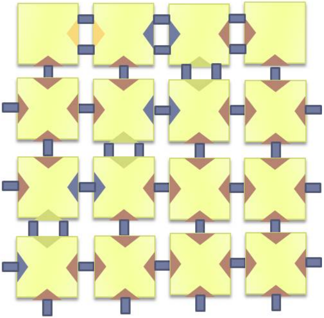


(Directed) tile complexity

Definition The minimum # of tile types necessary for a TAS at temperatures below τ to assemble S.

Notation $\mathrm{C}^{\mathrm{dtilec}(\leq\tau)}(S)$.

The above example shows that $\mathrm{C}^{\mathrm{dtilec}(\leq 2)}(Sq_n) \leq n + 3$, and it is actually $O(\log n / \log\log n)$ [Adleman et al. 2001].

Theorem ([Adleman et al. 2002])

It is NP-hard to compute directed tile complexity at the temperatures below 2.

But, for squares . . .

Minimum tile system for squares (MTSS)

INPUT a positive integer n

OUTPUT a smallest directed TAS ~~at a temperature below τ~~ that assembles the $n \times n$ square Sq_n.

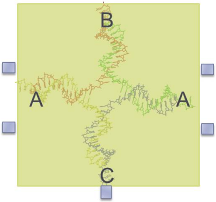
Polynomial-time algorithm for MTSS
[Chen, Doty, and Seki 2011]

1. # of "(local) behaviors of TASs" is polynomial in n.
2. For each of them,
   2.1 check whether the behavior is valid or implementable as a behavior of TAS;
   2.2 if YES, then check whether it assembles Sq_n.

Local behavior of a tile in a TAS

Let $\mathcal{T} = (T, \sigma, g, \tau)$ be a TAS. The behavior of $t \in T$ (how its 4 sides cooperate for attachment) is completely described by a system of 16 τ-inequalities (inequalities whose RHS is τ).

If $\tau = 4$, then

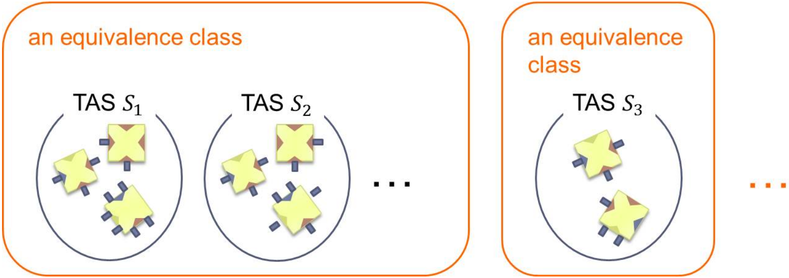


$g(\mathrm{A}) = 2, g(\mathrm{B}) = 0, g(\mathrm{C}) = 1$

$$\begin{cases} g(t(\mathtt{N})) & < & \tau \\ & \vdots & \\ g(t(\mathtt{W})) + g(t(\mathtt{E})) & \geq & \tau \\ & \vdots & \\ g(t(\mathtt{N})) + g(t(\mathtt{S})) + g(t(\mathtt{E})) & < & \tau \\ & \vdots & \\ g(t(\mathtt{N})) + g(t(\mathtt{W})) + g(t(\mathtt{S})) + g(t(\mathtt{E})) & \geq & \tau. \end{cases}$$

Local equivalence among TASs

Given TASs $\mathcal{T}_1 = (T, \sigma, g_1, \tau_1)$ and $\mathcal{T}_2 = (T, \sigma, g_2, \tau_2)$ with the same tile set T, they are locally equivalent if for each tile type $t \in T$, the induced systems of 16 inequalities are identical.



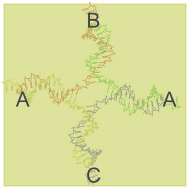
All TASs in a class behave exactly in the same way, and hence, output the same final product.

Validity or implementability of a behavior

Each equivalence class is represented by systems of 16 τ-inequalities assigned to tile types.

Some of such systems may not represent any class.

$$\begin{cases} g(t(\mathtt{N})) & < & \tau \\ g(t(\mathtt{W})) & \geq & \tau \\ g(t(\mathtt{S})) & < & \tau \\ g(t(\mathtt{E})) & < & \tau \\ g(t(\mathtt{W})) + g(t(\mathtt{E})) & \geq & \tau \\ & \vdots & \\ g(t(\mathtt{N})) + g(t(\mathtt{S})) + g(t(\mathtt{E})) & < & \tau \\ & \vdots & \\ g(t(\mathtt{N})) + g(t(\mathtt{W})) + g(t(\mathtt{S})) + g(t(\mathtt{E})) & \geq & \tau. \end{cases} \Rightarrow$$



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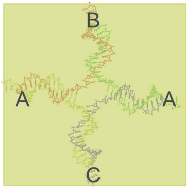


$g(A) \geq \tau$ and $g(A) < \tau$!?

This behavior is invalid (un-implementable).

FindStrength and FindOptimalStrength

FindStrength

INPUT a tile set T and systems of τ-inequalities;

OUTPUT a TAS whose tile set is T and each of whose tile types behaves as specified, if any.

There is a polynomial time algorithm for FindStrength [Chen, Doty, and Seki 2011].

FindOptimalStrength

Can we modify FindStrength so as to optimize the output TAS w.r.t. temperature in a polynomial time?

Main contribution

FindOptimalStrength is $\mathtt{NP}$-hard.

Threshold programming (TP)

GIVEN integer matrices C_1, C_2;

MINIMIZE τ;

SUBJECT TO $C_1 x \geq \tau \overrightarrow{1}$ and $C_2 x < \tau \overrightarrow{1}$, where x is a vector of positive integer variables.

Our special interest lies in TPs all of whose conditions are τ-inequalities of at most 4 terms like $v_1 + v_2 + v_3 < \tau$.

- Whenever TP is referred to, we assume this.
- This is for the reduction from TP to FINDOPTIMALSTRENGTH.

Karp-reduction from 1-in-3-SAT to FindOptimalStrength

The reduction proceeds $1 \to 2 \to 3 \to 4$.

1. Quadripartite 1-in-3-SAT;
2. τ-TP for any $\tau \geq 4$;
3. TP;
4. FindOptimalStrength.

Quadripartite 1-in-3-SAT is a variant of 1-in-3-SAT by Schaefer [Schaefer 1978], in which the variable set can be split into disjoint 4 subsets such that each clause contains at most 1 variable from each set. This is NP-hard even if no clause contains negated literals.

Karp-reduction from 1-in-3-SAT to 4-TP and to TP

τ-TP

Given integer matrices C_1, C_2, decide whether there is a positive integer vector x such that $C_1 x \geq \tau\vec{1}$ and $C_2 x < \tau\vec{1}$.

For $\tau = 4$, a clause $\{x_1, x_2, x_3\}$ is converted into

$$\begin{aligned} v_1 + v_2 + v_3 &\geq 4 \\ v_1 + v_2, v_1 + v_3, v_2 + v_3 &< 4. \end{aligned}$$

Theorem

4-TP is $\mathtt{NP}$*-hard. More strongly, for any* $\tau \geq 4$*,* τ*-TP is* $\mathtt{NP}$*-hard.*

Theorem

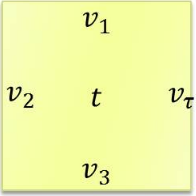
τ*-TP is* $\mathtt{NP}$*-hard.*

From TP to FindOptimalStrength

The following inequalities, which correspond to a clause of 1-in-3-SAT

$$\begin{aligned} v_1 + v_2 + v_3 &\geq 4 \\ v_1 + v_2, v_1 + v_3, v_2 + v_3 &< 4. \end{aligned}$$

are converted into a right tile type with the cooperation set below.



$$\begin{aligned} v_1 + v_2 &< \tau \\ v_2 + v_3 &< \tau \\ v_3 + v_1 &< \tau \\ v_1 + v_2 + v_3 &\geq \tau \\ v_\tau &\geq \tau. \end{aligned}$$

From TP to FindOptimalStrength (cont.)

Theorem

FindOptimalStrength *is* NP*-hard.*

Other results

Theorem

For any $\tau \geq 2$, there is a shape S_τ with $\mathrm{C}^{\mathrm{dtilec}(<\tau)}(S_\tau) > \mathrm{C}^{\mathrm{dtilec}(\tau)}(S_\tau)$.

Theorem

For any $\tau \geq 4$, it is $\mathtt{NP}$-hard to compute directed tile complexity at the temperatures below τ.

Thank you very much for your attention.

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Aalto University

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